Last name ______________________

First name ______________________

LARSON—MATH 310—CLASSROOM WORKSHEET 13
Null Space & Row echelon form

Review

- What is a *symmetric* matrix?
- Let A be any matrix. Why is $A^T A$ a symmetric matrix?
- Let A be any matrix. Why is AA^T a symmetric matrix?
- What is the *column space* $C(A)$ of a matrix A?
- What is the *row space* $C(A^T)$ of a matrix A?
- Describe the row space $C(A^T)$ of matrix A.

1. Can you find a "nice" description of $C(A^T)$, where $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 4 \end{bmatrix}$?

2. Find a specific (non-trivial) vector $\vec{v}$ in the row space of A.

   The *null space* $N(A)$ of a matrix A is the set of all vectors $\vec{x}$ where $A\vec{x} = \vec{0}$.

3. Find $N(A)$ by solving $A\vec{x} = \vec{0}$.

4. Find a specific (non-trivial) vector $\vec{x}$ in the null space of A.

5. Check that $\vec{v} \cdot \vec{x} = 0$.

6. Can you find a "nice" description of $N(A)$?

Row-reduced echelon form (RREF) of a matrix will be a tool to help us give the complete description of the solutions of a system of linear equations. We will use our row operations to get as many columns as possible with all zeros above and below a non-zero *pivot*. To standardize, we go from left column to right column.

Call the reduced matrix R. It will consist of pivot columns and "free" columns. The *rank* of A is the number of pivots of A.

Let

$$A = \begin{bmatrix} 1 & 1 & 2 & 4 \\ 1 & 2 & 2 & 5 \\ 1 & 3 & 2 & 6 \end{bmatrix}.$$

We used the above procedure to reduce A to the RREF

$$R = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

7. What is the rank of A?

8. Write the pivot columns.

9. Explain why the pivot columns are linearly independent.

10. Write the free columns.

11. Show that each free column is a linear combination of pivot columns.