

Last name \_\_\_\_\_

First name \_\_\_\_\_

LARSON—MATH 310—CLASSROOM WORKSHEET 12  
Column Space, Row Space, Null Space

Review

- The *transpose* of an  $m \times n$  matrix  $A = [a_{i,j}]$  is the  $n \times m$  matrix  $A^T = [a_{j,i}^t]$  where  $a_{j,i}^t = a_{i,j}$ .
- For *any* matrix  $A$ ,  $A^T A$  and  $AA^T$  are *square* matrices.

1. What is a *symmetric* matrix?
2. Let  $A$  be any matrix. Why is  $A^T A$  a symmetric matrix?
3. Let  $A$  be any matrix. Why is  $AA^T$  a symmetric matrix?
4. What is the *column space*  $C(A)$  of a matrix  $A$ ?
5. Describe the column space  $C(I)$  of  $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
6. Describe the column space  $C(A)$  of  $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ . Can you find a vector that is not in the column space?

7. Describe the column space  $C(A)$  of  $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 4 \end{bmatrix}$ .

8. Can you find a “nice” description of  $C(A)$ ?

The *row space*  $C(A^T)$  of a matrix  $A$  is the set of all linear combinations of its rows.

9. Describe the row space  $C(A^T)$  of matrix  $A$ .

10. Can you find a vector that is not in the row space of  $A$ ?

11. Can you find a “nice” description of  $C(A^T)$ ?

12. Find a specific (non-trivial) vector  $\vec{v}$  in the row space of  $A$ .

The *null space*  $N(A)$  of a matrix  $A$  is the set of all vectors  $\vec{x}$  where  $A\vec{x} = \vec{0}$ .

13. Find  $N(A)$  by solving  $A\vec{x} = \vec{0}$ .

14. Can you find a “nice” description of  $N(A)$ ?