Last name _____

First name _____

LARSON—MATH 310—CLASSROOM WORKSHEET 11
Transposes

Review

- How can you find the inverse of an invertible matrix?
- How can you check if a matrix is not invertible?
- What is a *lower triangular* matrix L?
- How can we find an LU factorization of a square matrix A?
- **Fact:** The product of lower-triangular matrices is lower-triangular.

The *transpose* of an $m \times n$ matrix $A = [a_{i,j}]$ is the $n \times m$ matrix $A^T = [a^t_{j,i}]$ where $a^t_{j,i} = a_{i,j}$.

1. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$. Find A^T.

2. Let $\vec{v} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$. Find $\vec{v}^T$.

3. Find $\vec{v}^T A^T$.

4. Find $A\vec{v}$.

5. Find $(A\vec{v})^T$.

6. Check that $\vec{v}^T A^T = (A\vec{v})^T$

7. Check that $\vec{v}^T A^T = (A\vec{v})^T$

8. Find $(A^T)^{-1}$.

9. Find A^{-1}.

10. Find $(A^{-1})^T$.

11. Check that $(A^T)^{-1} = (A^{-1})^T$.

12. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$. Find A^T.

13. Find $A^T A$.

14. Find AA^T.

15. What do you notice about both $A^T A$ and AA^T? Can you explain it?