Last name ________________________

First name ________________________

LARSON—MATH 310—CLASSROOM WORKSHEET 09
LU factorization

Review

- What is the *inverse* of a (square) matrix A?
- What is the *notation* for the inverse of a matrix A?
- When is a matrix A *invertible*?
- When is a matrix A *singular*?

Fact: The product of lower-triangular matrices is lower-triangular.

1. Let $L_1 = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$ and $L_2 = \begin{bmatrix} 4 & 0 \\ 5 & 6 \end{bmatrix}$. Find $L_1 L_2$ and check that it is lower-triangular.

2. Let $L_1 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$ and $L_2 = \begin{bmatrix} -1 & 0 & 0 \\ 5 & 6 & 0 \\ 1 & 2 & 3 \end{bmatrix}$. Find $L_1 L_2$ and check that it is lower-triangular.

Fact: The inverse of an (invertible) lower-triangular matrices is lower-triangular.

3. Find the inverse of the lower-triangular matrix $L_1 = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$.

An LU-factorization of a matrix A is a lower-triangular matrix L and an upper-triangular matrix U so that

$$A = LU.$$

The above facts—and elimination matrices—are what we need to show that any matrix that can be reduced to upper-triangular form *without row switches* admits an LU-factorization.

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

We will find an LU-factorization of A, that is, we will find a lower-triangular matrix L and an upper-triangular matrix U so that

$$A = LU.$$

4. Find an elimination matrix E corresponding to a row operation that "puts" a 0 in the A_{21} spot. Then find EA.

in

So EA is upper-triangular. Let $EA = U$.

5. Find E^{-1}.

Then $A = E^{-1}U$, where E^{-1} is lower-triangular and U is upper-triangular.

6. Let $L = E^{-1}$. Check that $A = LU$.