Last name _____

First name _____

LARSON—MATH 310—CLASSROOM WORKSHEET 04
Matrix times a Vector

Review

- Check that if the angle between $\vec{v}$ and $\vec{w}$ is θ then $\cos\theta = \dfrac{\vec{v}}{\|\vec{v}\|} \cdot \dfrac{\vec{w}}{\|\vec{w}\|}$.
- Check that if $\vec{v}$ and $\vec{w}$ are perpendicular then $\vec{v} \cdot \vec{w} = 0$.
- Find a (non-trivial) vector $\vec{u}$ which is perpendicular to $\vec{v}$.
- **Cauchy's Inequality**. For any vectors $\vec{v}$, $\vec{w}$, $|\vec{v} \cdot \vec{w}| \leq \|\vec{v}\|\|\vec{w}\|$.
- **Triangle Inequality** : $\|\vec{v} + \vec{w}\| \leq \|\vec{v}\| + \|\vec{w}\|$.

1. What is the "linear combination of columns" definition of a $m \times n$ matrix A times a vector x in $\mathbb{R}^n$?

2. Find $\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, using the "linear combination of columns" definition.

3. What is the "dot product with rows" definition of a $m \times n$ matrix A times a vector x in $\mathbb{R}^n$?

4. Find $\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, using the "dot product with rows" definition.

5. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$, $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. Solve $A\vec{x} = \vec{b}$.

6. How can we check if 3 vectors in $\mathbb{R}^3$ are co-planar?

Let $\vec{u} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$.

7. Are $\vec{u}$, $\vec{v}$, and $\vec{w}$ co-planar? Prove it by using our test.

8. What does it mean for vectors to be *linearly independent*?

9. Are $\vec{u}$, $\vec{v}$, and $\vec{w}$ linearly independent or linearly dependent?