

Last name \_\_\_\_\_

First name \_\_\_\_\_

LARSON—MATH 310—CLASSROOM WORKSHEET 03  
Dot Products

Review

- What is  $\vec{v} \cdot \vec{w}$ ?
- What is  $\|\vec{v}\|$ ?
- What is a *unit* vector?
- How can you find a unit vector in the direction of  $\vec{v}$ ?
- Check that if  $\vec{u}$  is a unit vector then  $\vec{u} \cdot \vec{u} = 1$ .
- Check that if  $\vec{v}$  and  $\vec{w}$  are vectors that point in the same direction then  $\vec{v} \cdot \vec{w} = \|\vec{v}\| \cdot \|\vec{w}\|$ .

1. Check that if the angle between  $\vec{v}$  and  $\vec{w}$  is  $\theta$  then  $\cos \theta = \frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\| \|\vec{w}\|}$ .

2. Check that if  $\vec{v}$  and  $\vec{w}$  are perpendicular then  $\vec{v} \cdot \vec{w} = 0$ .

Let  $\vec{v} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$  and  $\vec{w} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ .

3. Let  $\theta$  be the angle between  $\vec{v}$  and  $\vec{w}$ . Find  $\cos \theta$ .

4. Find  $\theta$ .

5. Find a (non-trivial) vector  $\vec{u}$  which is perpendicular to  $\vec{v}$ .

**Cauchy's Inequality.** For any vectors  $\vec{v}$ ,  $\vec{w}$ ,  $\vec{v} \cdot \vec{w} \leq \|\vec{v}\| \|\vec{w}\|$ .

6. Check that Cauchy's Inequality holds for vectors  $\vec{v}$ ,  $\vec{w}$ .

7. *Why* is Cauchy's Inequality true?

**Triangle Inequality :**  $\|\vec{v} + \vec{w}\| \leq \|\vec{v}\| + \|\vec{w}\|$ .

8. Check that the Triangle Inequality holds for vectors  $\vec{v}$ ,  $\vec{w}$ .

9. *Why* is the Triangle Inequality true?