# SAVING MATHEMATICS—FOR US (AND U.S.)

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The administration of West Virginia University (WVU) recently cut their graduate math programs. It is no longer possible to get a mathematics Ph.D. in the state of West Virginia. None of the university's president, its board, or the state's legislators are concerned about losing these programs. But they should be. The thought seems to be that mathematical *research* is not important (or at least not as important to keep as the majors like business that are currently popular at WVU; the state only needs to teach math service courses for whatever mathematics is needed in other disciplines (engineering students need calculus, computer science students need discrete mathematics, etc).

There are many reasons citizens might agree with the administration at WVU. I am a mathematician at a research university. When I tell people what I do, and mention that I do research as part of my job, people are often surprised (even shocked) that there is anything left to investigate. It is true that the mathematics that most people will see in school was created a long long time ago: all of the geometry that is studied was known by the ancient Greeks, all the algebra was known by the $15^{th}$ century, and calculus was invented in the $17^{th}$ century. What's left to do? It turns out that there are lots of unsolved mathematical problems, and they are not esoteric. Computers were invented in the $20^{th}$ century (largely by mathematicians) and to get them to compute many things of interest to us required the development of new *algorithms* (recipes for calculating numbers). Some of the mathematics created in the $20^{th}$ century is useful in our everyday lives. And in fact its omni-present. And mathematics has been central to our national defense (the GPS system, code breaking, weapons modeling, artificial intelligence, encrypting our communications, etc). And we will count on future mathematical developments for our happiness, productivity, defense, and even to make daily life possible.

I will elaborate on all of these things but, more importantly, I will explain why cutting mathematics graduate programs is short-sighted:

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mathematical results come from a broad array of researchers—of different abilities—that exist because of a culture that creates and develops mathematicians. Some are geniuses whose discoveries will go far beyond what we can imagine, while the majority will be average—but all are needed in order to reap the rewards mathematics research gives us. Mathematics is, in an important sense, like baseball. We don't have amazing players like Babe Ruth, Hank Aaron and Shohei Ohtani out of the blue. What we have are lots of kids playing baseball, lots of leagues, at all levels of development and ability, and finally a very top league where the best players go, and where the very best players will ultimately emerge. In order to have the top league at the level that it is, all of the lesser leagues make substantive contributions.

Mathematicians often revel in the supposed uselessness of what they do. The famous early $20^{th}$ century mathematician G. H. Hardy wrote: "I have never done anything 'useful'. No discovery of mine has made, or is likely to make, directly or indirectly, for good or ill, the least difference to the amenity of the world." This was mostly a dodge: he didn't see himself actually attempting to contribute anything of use outside of mathematics—and probably thought he couldn't had he been directed to work on anything obviously useful. Most mathematicians likely share Hardy's view—and most of the great (and useful) contributions of mathematics were likely done without any intention or hope of having the impact they had. Mathematicians will say they are interested in beauty. It is true that mathematical research is often a pursuit of patterns that strike a person's imagination: why do the primes appear among the natural numbers in the way that they do? And patterns suggest an analogy with art. It is perfectly consistent for mathematicians to have the self-image that they have as being useless pattern-seekers—while also doing enormously useful work. In fact though if mathematics weren't ultimately useful (outside of mathematics itself) it would not be studied (or at least no widespread public support for its study). There are no departments for instance of Purely Metaphysical Studies (where faculty research whether two angels can be in the same place at the same time). Why? While it is imaginable that someone might be interested in such questions (and in fact some philosophers have been interested in this question about angels), there is no history of the utility of these investigations. If there were a history of the usefulness of these investigations there would certainly be departments like this. In contrast, there is not only lots of mathematics from the distant past that we use today, but also lots of mathematics from the recent past.

Contributions of $20^{th}$ century mathematics to our lives include, for instance, the development of error-correcting codes that make message transmission possible. When you talk on a cell phone, or text, or interact with a web page, your words and clicks are transmitted as 0s and 1s from your phone to other computers around the world. These 0s and 1s can easily be corrupted: if an electrical burst changes a 0 to a 1 then the message is no longer the same. Coding theorists have spent considerable effort producing codes that can effectively minimize these errors. Better and better codes mean that our cell phones require less and less battery power in order to code and decode our voices and words. Better and better codes mean we can now transmit video (also 0s and 1s) across our solar system. The cell phone in your pocket is a computer. These were invented and developed by mathematicians. The path to the power of current computers is not only hardware innovations—but also *algorithmic* innovations. Algorithms are the recipes computers use to calculate, for instance, numbers. In order to digitize a voice, a computer uses an algorithm that translates vocal wave data to 0s and 1s. In many cases, the speed that computations have improved has as much or more to do with algorithmic improvements as hardware improvements.

A common problem is to allocate resources subject to a variety of constraints with the goal of minimizing a corresponding cost. A simple problem is to determine what quantities of foods need to be eaten in order to provide specified nutrient and calorie requirements while minimizing the total cost of the diet. Fast algorithms for solving these kinds of problems were developed by military planners during WWII. The corresponding mathematical discipline of linear optimization is now central in supply chain management and scheduling. Almost every Fortune 500 company uses linear optimization in their decision making. Any non-trivial scheduling problem, such as scheduling airline routes, requires linear optimization. While linear algebra (the theory of matrices) is a classical branch of mathematics, $20^{th}$ century results have provided new tools for data analysis—*principle component analysis* (PCA) is central to the investigation of the large data sets that are now ubiquitous, and closely related tools are used for a variety of practical problems including image and audio compression (removing redundancy from massive image files). Linear algebra also underlies web search algorithms (like Google's PageRank): the web can be thought of a a giant matrix indexed by the web pages themselves, or terms they contain, or other features, and whether or not one links to another—matrix methods can then be used to estimate the "importance" of a specific page for a user's search term. Differential equations are used to model a huge variety of phenomenon, including physical phenomenon like the weather, and how neutrons in a uranium pile will behave in energy development or weapons designs. This is another classical subject that requires $20^{th}$ century tools: these systems of equations can't be solved with classical tools, rather solutions are approximated using modern techniques (such as the *finite element method*). These examples of $20^{th}$ century mathematics that we use every day can be easily multiplied. There is every reason to expect that $21^{st}$ century mathematical contributions will continue to impact our lives—assuming there are similar conditions and support for this research.

Maybe administrators at WVU think that the mathematicians in their own department haven't been making these important contributions? Maybe they are right that mathematics can be cut at WVU and nothing would be lost—none of this ultimately useful mathematics has been or will be created at WVU, right? Maybe these administrators are right in the sense that there isn't a single area of mathematical research that was initiated at WVU, and maybe there is no famous theorem proved by a WVU mathematician, and maybe no one from WVU has ever won any of the famous mathematical prizes? Maybe no one can say that our material lives would be concretely different if it weren't for WVU mathematics research? These can all be true and it still be essential to have research mathematics departments like WVUs.

If it were true that no WVU mathematician has directly (obviously) advanced our material prosperity (or materially contributed to the defense of the country, etc) that would be akin to the fact that no WVU baseball player has ever been a Major League Baseball all-star. Its no more significant that no Babe Ruth played baseball for WVU than it is that no famous theorems were produced by WVU mathematicians. WVU baseball is one small part of the vast culture of baseball-playing kids, and youth leagues, high school baseball teams, minor league baseball teams, foreign baseball leagues, where players challenge each other to be better, and all of the coaches in those leagues that identify talent, help players improve their skills. Without this large culture very few if any Babe Ruths could actually make it to the big leagues: they either would not have played baseball (because there were fewer opportunities to play), or wouldn't have been identified, or wouldn't have been coached to hone their potential. In order for a Babe Ruth to make it to the big leagues, a vast network of non-Babe Ruths is necessary. You can't have Babe Ruths without having a system that also supports thousands and thousands of enthusiastic, hard-working, but average, baseball players.

And if we want to cultivate top-quality mathematical research we need a broad system to identify and nurture mathematical research talent. WVU is part of that system. As we need to support Average Joes in order to have Ruths, Aarons and Ohtanis, we need to support mathematical research at all of the WVUs across the country, and all the levels of mathematical community below them (in places like Eastern Europe there are even weekly after-school meetings—Math Circles—where interested students from $3^{rd}$ grade on can learn about mathematical topics that are outside their normal curriculum). Supporting a vast network of mathematicians and mathematical research departments is the way for us to produce the mathematics that will change our world. WVU's President Gee—and all similarly positioned decision-makers—should understand that a vibrant mathematical culture is required for continued mathematical research success—successes that have had huge impacts on our lives and successes that we hope will have similar future impact. WVU doesn't have to be MIT in order to importantly contribute to this project.

We need more than a few top schools where the very best young mathematicians tend to be attracted to. Useful mathematics is produced at every mathematical research department. And more importantly these "minor league" departments do sometimes themselves nurture and produce mathematicians that make recognizable contributions. My own area of research is *graph theory* (the study of networks, like airline routes, or social networks). Neil Robertson, one of the most famous researcher in my field went to little-known Brandon College (Canada). Graph theory research of the last 40 years has been shaped by his investigations. Robertson did get his Ph.D. at a top university (Waterloo, in Canada) but maybe he would never have been successful and made the contributions he made without the education and nurturing he received at Brandon?

The coach of WVU's baseball team might not obviously contribute to the quality of major league baseball—but he might. If he shows a raw fire-throwing pitcher with major league potential how to throw an effective curveball he has played an role. What Dr Gee and WVU administrators may not realize is that the major mathematical developments that are of everyday use in our lives are never ever fully formed; they are always improved and modified by others. While there are certainly mathematical geniuses, many important contributions are made by others as they absorb and understand recent work. In the case of solving linear optimization programs, one genius did invent a very fast algorithm (Dantzig's invention of the *simplex method*) but this idea had to be implemented—and could be improved. Mathematicians in fact have been continuously improving *solvers* (computer programs that produce linear optimization solutions) to make it possible to solve larger and larger systems faster and faster. They have also written solvers that can be easily used by the rest of us—making it possible for a wide variety of scientists and others to take advantage of these improvements.

When we don't support our mathematical culture mathematics talent gets wasted—and talent in math-adjacent fields. We are producing only a fraction of the engineers (per capita) of countries like China. Presumably the same fraction of U.S. kids have the same high-level abilities as Chinese kids. If our smart kids predominantly end up in finance (for instance), we are destined to fall behind in other technical disciplines. We have enormous problems on the horizon that, it is reasonable to assume, will need new mathematical tools. North Korea currently leads the world in cyber-currency theft, quantum computers will require us to develop new encryption tools in order to safeguard our banking and financial systems, we are in an artificial intelligence arms race with China, and we will need to better model a large variety of systems in order to improve our future prospects (for instance new materials and new battery and energy technologies).

Finally, it is worth mentioning that it doesn't cost much to get the benefits of a research mathematics culture. Mathematicians typically get paid average salaries at best: faculty in other departments often get paid twice as much as mathematics faculty. President Gee of WVU has reportedly spent millions of dollars on private air travel: savings on luxury travel like this might go a long way to pay for research mathematics at WVU. He has been extraordinarily short-sighted: he projected WVU's student body to go up by 25%—while demographers have been reporting a coming drop in college-age students (the "demographic cliff"). And he undertook extraordinary new construction projects. Attendance did drop—but all the new buildings must still be paid for. A better use for these resources is building a stronger mathematical—and mathematical research—culture. West Virginia needs it, we need it—and the country needs it. WVU and every other educational institution needs vision, vision to look beyond the popularity of current popular majors, vision to express to students and educators that mathematics is not only foundational, but that mathematical research is important, and vision to think more than one step ahead.

In order to face the future's challenges we'll need to think ahead and, most likely, we'll need and use newly developed mathematical tools.

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