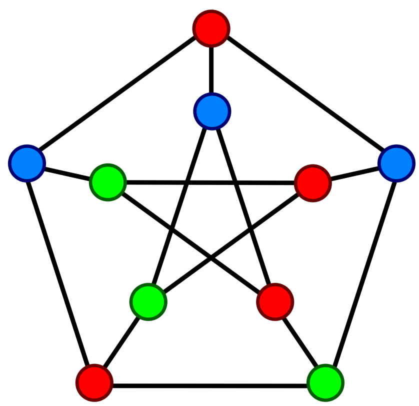
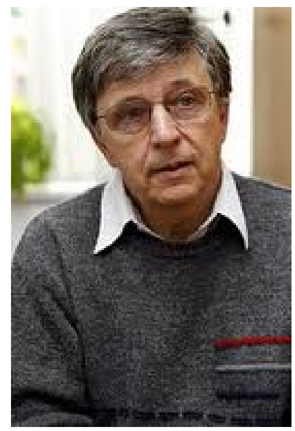
# VCU Math   Fall 2026   *MATH 756*

# **Orthogonal Graph Representations, Lovász's Theta and Independent Sets**

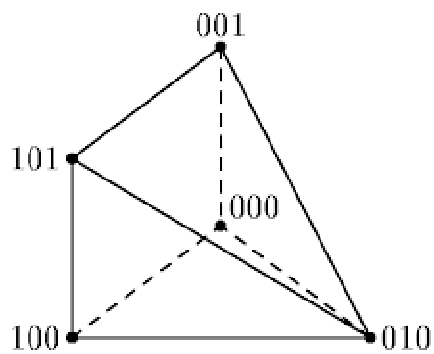
Prof Craig Larson

MW, 9:30 - 10:45 a.m.





$$\vartheta$$




An *orthonormal representation* of a graph is a set of unit vectors associated to vertices where non-adjacent vertices are orthogonal. Lovász's theta function is:

$$\vartheta = \min_{\{\hat{x}_i\},\hat{c}} \max_i \frac{1}{(\hat{c}\cdot\hat{x}_i)^2}$$

for any orthogonal representation $\{\hat{x}_i\}$ and any unit vector $\hat{c}$. This number can be computed efficiently. The Sandwich theorem says that is always between the NP-hard-to-compute independence number $\alpha$ and the cardinality of a minimum clique cover $\bar{\omega}$.

For perfect graphs $\alpha = \vartheta = \bar{\omega}$. An AT (alpha-theta) graph is a graph where $\alpha = \theta$. This class generalizes the perfect graphs, contains the Kőnig-Egerváry graphs (and the Petersen graph!), and shows up in quantum information theory. Not much is known about this class of graphs. What other graphs does this class contain? Can they be recognized efficiently. Can they be leveraged to speed up algorithms for computing $\alpha$? We will thoroughly investigate these questions.

For questions or more information, email Craig Larson (`clarson@vcu.edu`).